Multidimensional Equality of Opportunity in the United States*

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Abstract

Are the United States still a land of opportunity? We provide new insights on this question by invoking a novel measurement approach that allows us to target the joint distribution of income and wealth. We show that inequality of opportunity has increased by 56% over the time period 1983-2016. Increases are driven by two distinct forces: (i) a less opportunity-egalitarian distribution of income until 2000, and (ii) a less opportunity-egalitarian distribution of wealth after the financial crisis in 2008. In sum, our findings suggest that the US have consistently moved further away from a level playing field in recent decades.

JEL: D31, D63, J62
Keywords: Fairness, Intergenerational Mobility, Time Trends, Measurement

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1 INTRODUCTION

In a fair economy, people act on a level playing field to acquire monetary resources. This idea—oftentimes labeled as *equality of opportunity*—is widely reflected in fairness conceptions of academic philosophers and the general public (Alesina et al., 2018; Almås et al., 2020; Arneson, 2018; Cappelen et al., 2007; Cohen, 1989; Fong, 2001; Rawls, 1971; Roemer, 1998). As a consequence, there is an active literature in economics that assesses the satisfaction of the opportunity-egalitarian ideal in different countries at different points in time. We contribute to this literature by providing the first analysis of the association between family background characteristics and the joint distribution of income and wealth in the US.

Existing studies on inequality of opportunity and intergenerational mobility focus on income—and to a lesser extent on wealth—to measure monetary resources.[1] By excluding either income or wealth from the analysis, these studies neglect important information on individual consumption possibilities which arguably are the relevant metric to assess the financial well-being of individuals. For example, unidimensional analyses will misrepresent the financial well-being of income-poor heirs who support their lifestyle by selling assets and asset-poor persons with high incomes. Therefore, if society cares for the financial well-being of individuals more broadly, we should move from unidimensional analyses of monetary resources to analyses of the joint distribution of income and wealth.

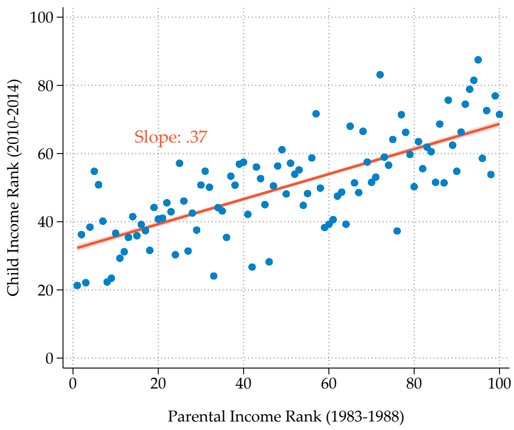
The focus on unidimensional analyses would be innocuous if income and wealth were perfect substitutes as indicators for monetary resources. There are at least two reasons why this is implausible. First, well-off parents transmit monetary resources to the next generation through bequests and inter vivo gifts (Boserup et al., 2016; Elinder et al., 2018; Wolff, 2002). In turn, expected wealth transfers distort the education and labor supply decisions of children (Kindermann et al., 2020; Kopczuk, 2013). Such behavioral responses create a wedge between the relative positions of individuals in income and wealth distributions: individuals that receive a lot of wealth from their parents are not necessarily those who earn high incomes. This observation is particularly relevant for the analysis of time trends as inheritances have grown in many Western societies in recent decades (Piketty and Zucman, 2015). Second, changes in wealth are a function

[1] For the US, see Chetty et al. (2014a), Davis and Mazumder (2017), and Solon (1992) for intergenerational income mobility; Charles and Hurst (2003) and Pfeffer and Killewald (2018) for wealth mobility, and Hufe et al. (2023), Niehues and Peichl (2014), and Pistolesi (2009) for inequality of opportunity in incomes.

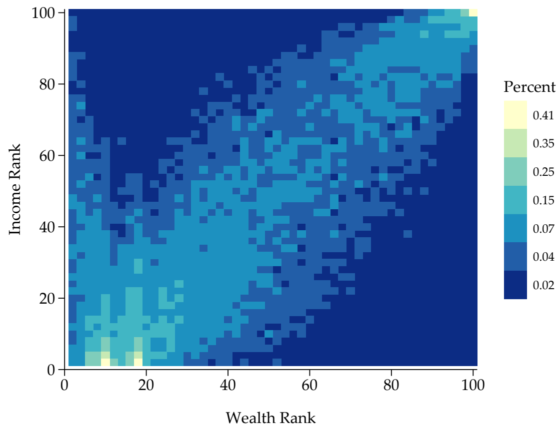
of savings and asset price changes. While the savings channel depends on income, the price channel depends on portfolio compositions. Therefore, changes in asset prices are another force that drives a wedge between the relative positions of individuals in income and wealth distributions. Again, this observation is particularly relevant for the analysis of time trends as wealth-to-income ratios—and therefore the sensitivity of wealth to asset price fluctuations—has grown over time (Kuhn et al., 2020).

In Figure 1, we use data from the Panel Study of Income Dynamics (PSID) to show that these concerns are relevant for the analysis of equal opportunities in the United States. In Panel (A), we replicate the well-known finding that child incomes increase with the income of their parents during childhood: an increase of parental income by 10 percentile ranks is associated with an average increase of 3.7 percentile ranks in child income. This estimate is very similar to the slope estimate of 0.34 in Chetty et al. (2014a). In Panel (B), a heatmap of income and wealth ranks demonstrates that income and wealth are far from perfect correlates (Rank correlation $\rho = 0.55$).[2] Taken together,

FIGURE 1. Intergenerational Income Mobility and the Distribution of Monetary Resources in the United States



(A) Income Transmission



(B) Joint Distribution of Income and Wealth

Data: PSID.
Note: Panel (A) shows a binned scatter plot of average child income ranks in the period 2010-2016 by income rank of their parents in the period 1983-1988. All individuals are aged 25-60. Panel (B) shows a heatmap of year-specific income and wealth ranks for the pooled sample of individuals aged 25-60 in the period 1983-2016. Each data point shows the share of individuals in a fixed two-percentile income (wealth) bin that belong to a particular two-percentile wealth (income) bin. See Section 3 for detailed definitions of income and wealth.

[2]The moderate rank correlation is not due to idiosyncratic fluctuations in income or wealth. Using 5-year moving averages for income and wealth yields $\rho = 0.59$.

these patterns suggest that unidimensional analyses of equality of opportunity and intergenerational mobility provide a distorted image of the importance of family background for individual consumption possibilities and financial well-being.

In this paper, we address these shortcomings by analyzing the association between family background and the joint distribution of income and wealth. We use the PSID to implement a novel measure of multidimensional equality of opportunity (Kobus et al., 2020). Our analysis proceeds in two steps. First, we construct an *intergenerational sample* in which we measure equality of opportunity in monetary resources by using parental income ranks as the only proxy for socioeconomic background. This practice is consistent with the literature on intergenerational mobility; however, the sparsity of data links across generations prevents meaningful analyses of time trends. Second, we construct an *individual sample* in which we substitute parental income ranks by a vector of alternative socioeconomic background characteristics. These data are available on an annual basis and allow us to assess trends over the period 1983-2016.

Our findings can be summarized as follows. First, multidimensional inequality of opportunity is consistently and substantially higher than inequality of opportunity in income. Hence, unidimensional analyses that focus on income only underestimate the extent to which monetary resources are associated with family background. Second, the playing field in the US has become more tilted in recent decades: inequality of opportunity in 2016 is 56% higher than in 1983. Furthermore, time trends are markedly different when accounting for the multidimensionality of monetary resources. For example, an exclusive focus on income suggests small increases in unequal opportunities after the year 2000. This relative stability, however, is accompanied by strong increases in the wealth dimension leading to an overall increase in unequal opportunities.

The contribution of this paper is threefold. First, we complement recent literature that characterizes the joint distribution of income and wealth in the US (Berman and Milanovic, 2020; Kuhn et al., 2020). This literature focuses on inequalities in outcomes but remains silent on opportunities and intergenerational transmission processes. Second, we provide novel insights regarding the development of equality of opportunity in the United States. While existing literature documented relative stability of equality of opportunity in terms of income after 2000 (Chetty et al., 2014b; Hartley et al., 2022), we show that decreases emerge once we account for the wealth dimension. Third, we provide a novel decomposition of the multidimensional measure into inequality of opportunity in income, inequality of opportunity in wealth, and the association of both outcomes across family background types. Association is a distinctive feature of joint distributions that cannot be captured by unidimensional analyses. It indicates whether individuals of a given family background are more likely to fare better or worse in both dimensions simultaneously. We use a multidimensional framework to combine these dimensions and to obtain an overall conclusion regarding the extent of unequal opportunities in the US.

2 MEASUREMENT

Consider a population $\mathcal{N} := \{1, ..., N\}$ and a set of outcomes $\mathcal{K} := \{1, \ldots, K\}$ that capture monetary resources. Individuals $i \in \mathcal{N}$ receive utility from $q \in \mathcal{K}$. We can summarize the distribution of monetary resources by outcome matrix X of dimension $N \times K$, where an element x_{iq} denotes i's outcome in dimension q. Outcomes are determined by two sets of factors: a set Ω that captures family background characteristics and a set Θ that captures individual choices. We define $\omega_i \in \Omega$ as a comprehensive description of family background and $\theta_i \in \Theta$ as a comprehensive description of the choices made by $i \in \mathcal{N}$. For each q, there is an outcome-generating function defined as follows:

$$x_{iq} = f_q(\omega_i, \theta_i), \ \forall i \in \mathcal{N}. \tag{1}$$

In an equal-opportunity society, outcome differences are determined by individual choices θ_i but are invariant to family background ω_i (Roemer, 1998). There are different ways of translating this idea into measures. Most empirical literature relies on an *ex-ante* approach, which broadly consists of two steps. First, one partitions the population into types $T = \{t_1, ..., t_M\}$. Individuals belong to a type if they share the same set of family background characteristics: $i, j \in t_m \Leftrightarrow \omega_i = \omega_j$. For example, in rank-rank measures of intergenerational mobility, types are defined by parental income ranks. Second, one assesses differences in average outcomes across types by regressing child outcomes on a measure of family background:

$$x_{iq} = \alpha_q + \beta_q \omega_i + \epsilon_{iq}. \tag{2}$$

There are two prominent ways of summarizing the resulting information in measures of inequality of opportunity: (i) β_q, which is the standard statistic in the literature on *intergenerational mobility* (Black and Devereux, 2011). (ii) $I(X) = I(\mathbb{E}[x_{iq}])$, where $I()$ is any inequality index, and which is the standard statistic in the literature on *equality of opportunity* (Roemer and Trannoy, 2016). It defines inequality of opportunity as inequality between types: all within-type variation is removed and inequality reflects only inequality that arises due to family background. Clearly, both measures are isomorphic and capture the opportunity-egalitarian idea: the higher β_q, the more life outcomes x_q are predicted by family background ω_i, and the higher the corresponding measure of inequality of opportunity.

In this paper, we follow the tradition of the equality of opportunity literature and summarize outcome differences across types with an inequality index. In particular, we use the measure of Kobus et al. (2020), which allows us to account for the multidimensionality of monetary resources. For the sake of simplicity and in line with our empirical application, we focus on the case of two outcome dimensions and set $K = 2$. In this case, the index is given by

$$I(X) = 1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \frac{(\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{(\mu_1)^{r_1} (\mu_2)^{r_2}} \right)^{\frac{1}{r_1 + r_2}} \qquad \forall_t \, a_t < 0, \; r_1, r_2 < 0, \tag{3}$$

where N_t denotes the number of individuals in type t and μ_q^t (μ_q) the type (population) means in outcome q. In the following, we will describe the roles of r_q and a_t which are weights for outcome dimension q and types t, respectively. However, before doing so we note that if $r_q = 0$ for either outcome, $I(X)$ boils down to a unidimensional measure of inequality of opportunity which is the well-known Atkinson (1970) index applied to types t.[3]

Dimension weights r_q govern the sensitivity of the measure to between-type inequality in outcome q. The more negative r_q, the more convex the measure in q, and the higher its sensitivity to between-type inequality in this dimension. For example, if $r_1 < r_2$, $I(X)$ is more sensitive to inequality in the first than in the second outcome. This outcome is then relatively more important in the inequality assessment. r_q is also related to the degree of inequality aversion ϵ_q via $r_q = 1 - \epsilon_q$. As ϵ_q rises, the index becomes more sensitive to inequality at the bottom of the distribution than at the top. Note that ϵ_q is a parameter chosen by the researcher. In his seminal work, Atkinson (1970) arbitrarily set ϵ_q equal to 1, 1.5 and 2. Subsequently, empirical research has tried to infer plausible values of ϵ_q from economic policy design and tax schedules (Aristei and Perugini, 2016; Gouveia and Strauss, 1994; Young, 1990). These estimates range between 1 and 2 depending on the country and period of interest. In our baseline calculations, we choose $\epsilon_q = 1.2$ ($r_q = -0.2$) for both income and wealth. However, in section 5, we

[3]Generally, the index in Equation (3) is a multidimensional generalized entropy measure. Similarly, the empirical literature on equality of opportunity often uses unidimensional generalized entropy measures like the mean log deviation to summarize inequality between family background types.

show that our conclusions on time trends do not change for a wide range of plausible choices for ϵ_q.

Type weights a_t determine how much the social planner values respective types. The lower a_t, the higher the weight attached to type t. To ensure that $I(X)$ measures inequality, higher weight is assigned to types that have lower values of $(\mu_1^t)^{r_1}(\mu_2^t)^{r_2}$. Note that a_t is also a parameter chosen by the researcher. In our benchmark calculations, we choose type weights that decrease linearly with type ranks in the values of $(\mu_1^t)^{r_1}(\mu_2^t)^{r_2}$. However, in section 5, we show that our conclusions on time trends do not change when type weights are concave or convex in these ranks.

The functional form $(\mu_1^t)^{r_1}(\mu_2^t)^{r_2}$ is the same as the Cobb-Douglas utility function but the parameterization of weights differs. In particular, both a_t and r_q are negative ensuring that the index is convex and supermodular. Convexity ensures sensitivity to Pigou-Dalton transfers between types, i.e., that $I(X)$ increases after transfers that increase between-type inequality in dimension q. This is a fundamental property for ex-ante measures of inequality of opportunity. Supermodularity ensures sensitivity to correlation-increasing transfers, i.e., that $I(X)$ increases after transfers that increase the cross-type correlation of income and wealth. This is a fundamental property for multidimensional measures of inequality. It is important to note that this functional form is not adopted arbitrarily but that it is derived from first principles: $I(X)$ is the only index fulfilling the fundamental principles of ex-ante equality of opportunity in a multidimensional setting while satisfying standard properties of inequality measures such as *monotonicity*, *utilitarian aggregation*, and *ratio scale invariance*.[4]

Beyond its normative foundations, the index has several useful properties for empirical analyses. First, it can be decomposed to distinguish between the impact of inequality of opportunity in each outcome and the association of outcomes across types. We will use this property in our empirical analyses to understand the drivers of inequality of opportunity in the US. Second, the index is bounded in the interval $[0, 1)$. If $\mu_q^t = \mu_q$ for every type t and outcome q, the index will be zero. Finally, the index is a welfare-based inequality measure in line with the pioneering work by Atkinson (1970). For example, a value of 0.25 (0.5) means that existing inequality of opportunity imposes a welfare cost of 25% (50%) of the population average of each outcome. In other words, if there was perfect equality of opportunity, society would achieve the same level of welfare using only 75% (50%) of the available monetary resources in income and wealth (Atkinson, 1970; Kolm, 1969; Sen, 1973).

[4]See also Supplementary Material A for a simple exemplary illustration of its core properties.

3 DATA

Data Source. We assess the evolution of equal opportunities in the US while accounting for the multidimensionality of monetary resources. Therefore, we require data with information on income, wealth, and family background characteristics that are available for a long time period. In the US, the Panel Study of Income Dynamics (PSID) is the only publicly available data source that satisfies these criteria. For example, while the Survey of Consumer Finances (SCF) offers a long time series on household income and wealth, it contains limited information on the family background of its respondents.

Since 1968 the PSID collects rich information on income and family background characteristics for a nationally representative sample of US households. Since 1984 it also collects data on wealth.[5] Children who leave the parental household become independent units in the PSID sampling frame. Therefore, it is possible to link data across generations.

Income information is collected for the year predating the survey year. Hence, we use information from the income reference (survey) period 1983-2016 (1984-2017). We now turn to a description of relevant variables.

Monetary Resources. We consider two dimensions of monetary resources: income and wealth. We measure income as annual disposable household income. It comprises total household income from labor, asset flows, windfall gains, private transfers, public transfers, private retirement income and social security pensions net of total household taxes. We measure wealth as household net worth. It comprises the sum of home equity, other real estate, private businesses, vehicles, transaction accounts, corporate equities, annuities/IRAs and other savings net of any debt.

We scale household incomes and wealth by the modified OECD equivalence scale. Hence, we measure both income and wealth at the household level, whereas the units of analysis are individuals. This choice is consistent with our overarching interest in consumption possibilities since the application of equivalence scales allows for resource sharing among household members.

[5] Until 1999 wealth information was collected every five years. Since then, it is a regular part of every PSID wave.

Wealth data in the PSID is often considered inferior to wealth data in the SCF. Therefore, we compare PSID and SCF concerning time trends in household net worth in Supplementary Figure S.1. Due to oversampling of wealthy households, the SCF assigns a larger share of total net worth to the top 10% of the wealth distribution. Yet, level differences at the top are the only notable difference between PSID and SCF. Importantly, time trends in household net worth are consistent across both data sources.[6]

Family Background Characteristics and Types. We consider two alternative ways to measure family background. First, we use parental income ranks for the total incomes of mothers and fathers averaged over the years 1983-1988. We residualize parental income from the first and second-order polynomials of parental age to account for life-cycle effects in parental earnings profiles. We then partition the population into 36 types by ranking total parental income. Second, we use a vector of alternative socioeconomic background variables. This vector includes parental education (3 categories), parental occupation (3 categories), race (2 categories), and Census region of upbringing (2 categories).[7] We partition the population into 36 types based on the combination of these family background variables.

Estimation Samples. We base our estimates on two different samples. First, we construct an *intergenerational sample* of $1,366$ individuals. To obtain this sample, we drop all individuals with (i) missing and negative income and wealth, and (ii) missing information on parental education, parental occupation, race and region of upbringing. Then we match all respondents to both of their parents and drop observations with (iii) missing parental income. Lastly, we restrict observations to children (parents) aged 25-60 in the period 2010-2016 (1983-1988).[8] This sample allows us to proxy ω with parental income rank, which is common practice in the literature on intergenerational mobility. However, it imposes restrictions on the analysis of time trends since one requires information on both parental and child outcomes while allowing for sufficient time between these observations.

[6]See also Pfeffer et al. (2016) for a detailed comparison of wealth definitions in PSID and SCF.

[7]Parental education: low (less than high school), intermediate (high school), high (some college and more); parental occupation based on 1-digit 1990 Census codes: low (4,8,9), intermediate (3,5,6,7), high (1,2); race: white (non-Hispanic), other; Census region of upbringing: South, other.

[8]Supplementary Table S.2 details how the different restrictions affect the final sample size.

Second, to investigate time trends, we construct an *individual sample*.[9] In contrast to the previous sample, we drop requirement (iii). Again, we limit the sample to individuals aged between 25-60. We obtain a sample of at least $4,000$ observations in every year of the period 1983-2016 which allows us to monitor the development of equality of opportunity in the US over 33 years.

We are conscious that the PSID is subject to selective survey attrition across waves and that our data restrictions may distort our sample through selective item non-response. Therefore, we follow Meyer et al. (2015) and perform all calculations using sampling weights that match the Current Population Survey (CPS). Results remain unchanged when using standard survey weights provided by the PSID instead. Descriptive statistics for all estimation samples are disclosed in Table S.1.

4 RESULTS

Our analysis proceeds in two steps. First, we measure equality of opportunity in the *intergenerational sample*. Thereby, we either use parental income ranks or the vector of alternative socioeconomic background variables to proxy for family background. We will show that both approaches yield very similar results. Second, having validated the use of alternative socioeconomic background variables, we use the *individual sample* to analyze trends in equality of opportunity in the period 1983–2016.

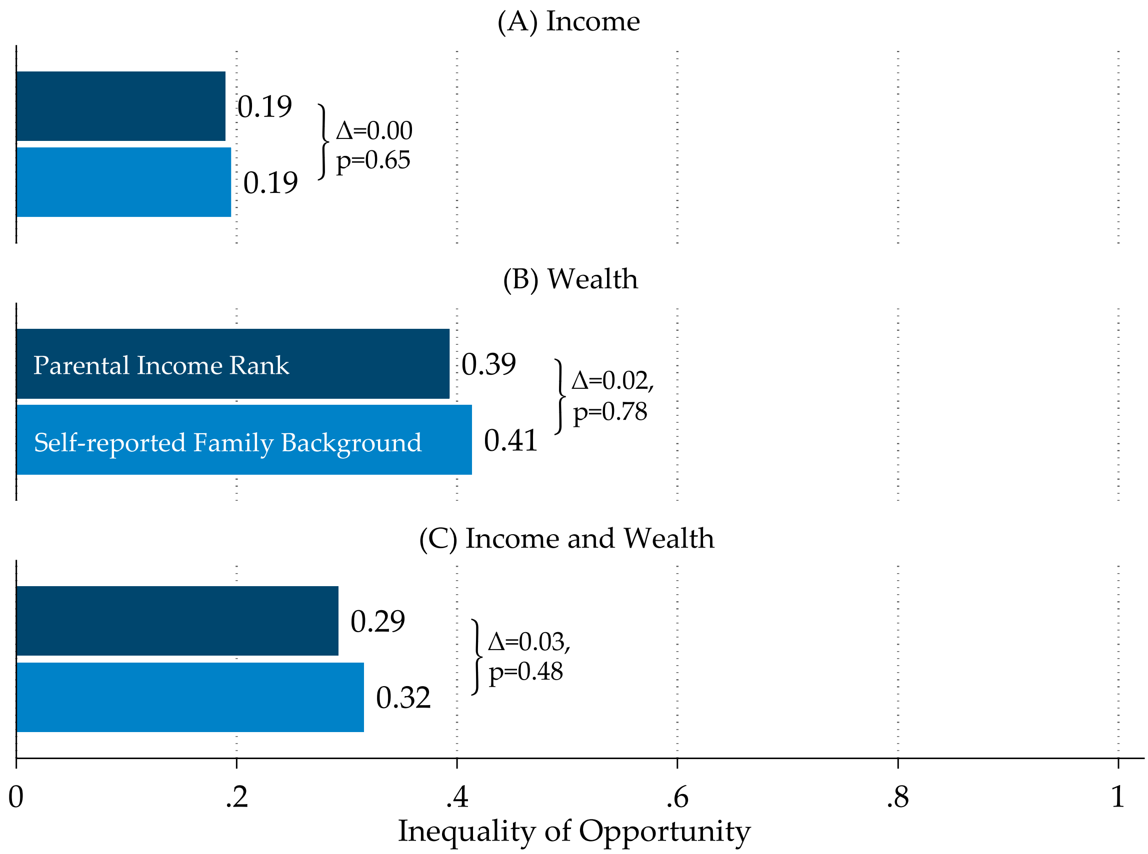
Intergenerational Estimates. Figure 2 shows estimates for inequality of opportunity in the *intergenerational sample* for different combinations of outcomes and family background variables.

First, we focus on the dark-blue bars that show estimates based on parental income ranks. In Panel (A), we measure monetary resources by income only and inequality of opportunity amounts to 0.19. In Panel (B), we measure monetary resources by wealth and inequality of opportunity doubles to a level of 0.39. Finally, in Panel (C) we account for the multidimensionality of monetary resources by considering both income and wealth. Then, inequality of opportunity amounts to 0.29. These results suggest that

[9] We note it is possible to analyze time trends in an intergenerational sample when focusing on income and wealth measures in early adulthood (e.g. Hartley et al., 2022). However, such an age restriction would not be adequate in our setting due to lifecycle gradients in both income and wealth.

we tend to underestimate tilt in the playing field when relying on income as the sole proxy for monetary resources.

FIGURE 2. Inequality of Opportunity in the US Intergenerational Sample



Data: PSID.
Note: This figure shows estimates of inequality of opportunity in the US for the *intergenerational sample*. Panel (A) (Panel [B]) shows results for a unidimensional definition of monetary resources based on income (wealth). Panel (C) shows results for a multidimensional definition of monetary resources based on income and wealth. In each panel, inequality of opportunity estimates are based on 36 types according to alternative definitions: parental income rank or self-reported family background. Estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$. Δ indicates the difference in inequality estimates across type definitions. p-values for the null hypothesis that $\Delta = 0$ are bootstrapped using 1,000 draws.

Second, we focus on a comparison between dark-blue bars and light-blue bars. To estimate the latter, we replace parental income ranks with a vector of alternative socioeconomic background characteristics. Point estimates remain virtually unchanged by this alternation and we cannot reject the equality of estimates at conventional levels of significance. This result suggests that parental income ranks and alternative socioeconomic background characteristics contain similar information about family background. In general, this is an encouraging message as data sets including intergenerational links are much scarcer than data sets including retrospective information on various socioeconomic background variables.[10]

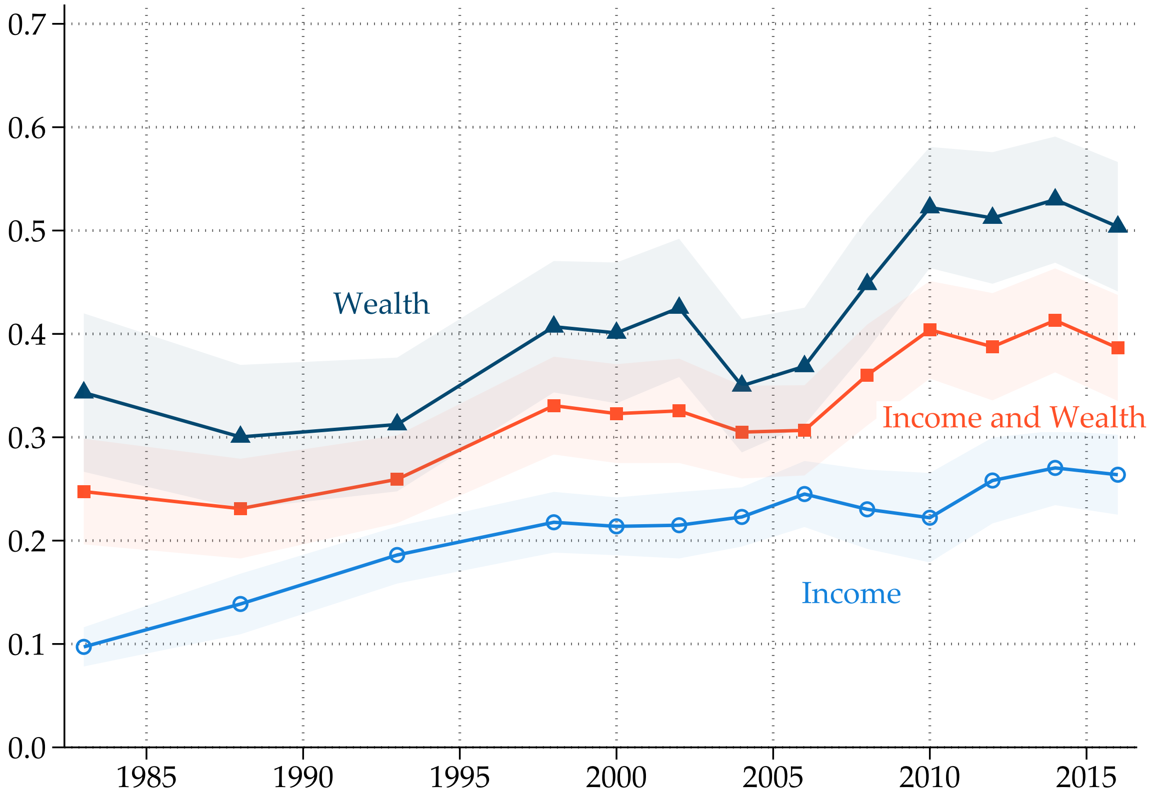
This conclusion is robust to a variety of checks. First, it is well-known that PSID subsamples with intergenerational links are positively selected on their socioeconomic status (Ward, 2021). Therefore, we re-weight the *intergenerational sample* to match the broader population characteristics concerning parental education, parental occupation, race, Census region of upbringing, and age. The re-weighting has little effect on inequality of opportunity estimates (Supplementary Figure S.3). Second, existing literature documents life-cycle bias in intergenerational mobility estimates. Estimates of both income and wealth mobility tend to be downward (upward) biased when children are young (old) (Haider and Solon, 2006; Mazumder, 2018; Nybom and Stuhler, 2016). This bias is usually addressed by measuring income in midlife. There are slight upward corrections of inequality of opportunity in wealth when we restrict our sample to the age range 40-45 (Supplementary Figure S.4). Importantly, however, differences in results based on income ranks and alternative socioeconomic background characteristics remain small for all considered age ranges. Third, we compare estimates based on the alternative background characteristics to expanded sets of family backgrounds where we add parental income and parental wealth ranks. The resulting estimates are very close to our baseline estimates suggesting that the vector of alternative socioeconomic background characteristics captures most of the relevant cross-family variation in socioeconomic status (Supplementary Figure S.5).

We conclude that the vector of alternative socioeconomic background characteristics provides suitable information to capture intergenerational disadvantage. As these data are available on an annual basis, we can use them to assess time trends in inequality of opportunity.

Time Trend (1983-2016). Figure 3 shows the development of inequality of opportunity in the US over the period 1983-2016. The following patterns emerge.

[10] Jácome et al. (2021) use a similar strategy and approximate parental income with self-reported background characteristics, i.e., retrospective information collected from the respondents of interest and not their parents. In Supplementary Figure S.2, we show that income distributions within family background types are broadly comparable regardless of whether we use types based on parental income ranks or self-reported background characteristics.

FIGURE 3. Inequality of Opportunity in the US, 1983-2016
Baseline Estimates



Data: PSID.
Note: This figure shows estimates of inequality of opportunity in the US for the *individual sample* over the period 1983-2016. Inequality of opportunity estimates are based on 36 types according to the following socioeconomic background characteristics: parental education, parental occupation, race, and region of upbringing. Estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$. 95% confidence intervals (shaded areas) are bootstrapped using 1,000 draws.

First, inequality of opportunity in income increased from 0.10 to 0.26 over time. We can distinguish two distinct periods. On the one hand, we observe marked increases from 1983 to 1998. On the other hand, there are only moderate increases after the year 2000. This two-partite pattern is consistent with findings from the literature on intergenerational income mobility. For example, Davis and Mazumder (2017) show that equality of opportunity decreased for cohorts born in the 1960s and that entered the labor market after 1980. Chetty et al. (2014b) show that this trend flattens for cohorts born in the 1970s that enter labor markets in the 1990s and 2000s. Likewise, Hartley et al. (2022) document a flat time trend in the intergenerational income correlation of mothers and daughters after 2000.

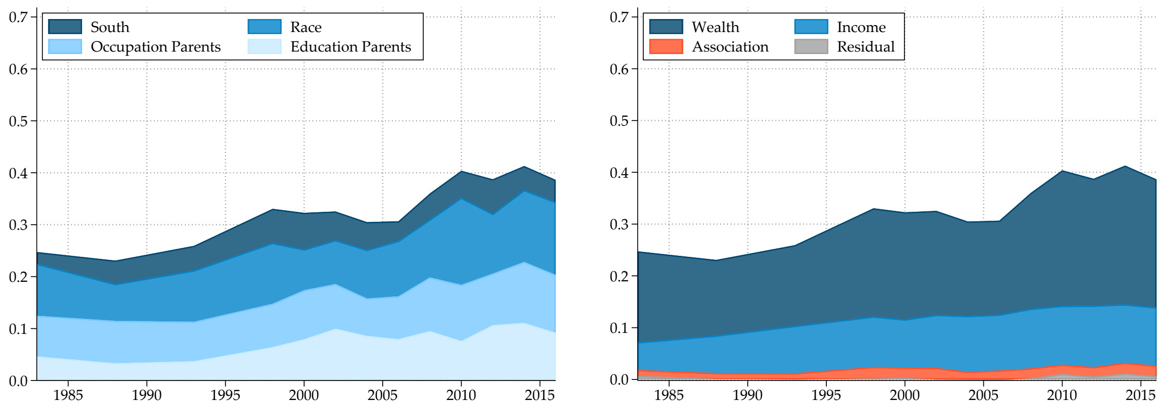
Second, inequality of opportunity in wealth increased from 0.34 to 0.50 over time. Again, we can distinguish two distinct periods. On the one hand, we observe moderate increases from 1983 to 2006. In these years, increases in the stock market were accompanied by a robust housing market (Kuhn et al., 2020; Wolff, 2017). Since owner-occupied housing has a higher weight in the portfolios of individuals from lower socioeconomic backgrounds, increasing house prices attenuated the tendency towards a less opportunity-egalitarian distribution of wealth. On the other hand, differences in portfolio compositions across socioeconomic backgrounds started working in the opposite direction with the financial crisis in 2008. While the stock market experienced a quick recovery, house prices did not catch up to their pre-crisis level. As a consequence, the wealth distribution has become less opportunity-egalitarian with the crisis—a trend that has not reverted ever since.

Taken together, the playing field for the acquisition of monetary resources has become more tilted over time. Starting at a level of 0.25 in 1983, inequality of opportunity in the joint distribution of income and wealth reached a level of 0.39 in the latest period of observation. This shift corresponds to an increase of 56%. Importantly, the trend towards decreasing opportunities to acquire monetary resources continues after the year 2000. This finding can be related to extant literature invoking intergenerational income mobility estimates to conclude relative stability in equality of opportunity in recent years (Chetty et al., 2014b; Hartley et al., 2022). To the extent that these works aim to proxy financial opportunities more generally, they miss important information by focusing on income only. When accounting for the multidimensionality of monetary resources, one cannot reject the claim that opportunities in the US have declined after the year 2000.

Decomposition. To develop a better understanding of these trends, we conduct a Shapley value decomposition, i.e., we decompose the trend in equality of opportunity into the contributions from different socioeconomic background characteristics: parental education, parental occupation, race, and the region of upbringing (Figure 4, Panel [A]).

First, 57% of the overall increase in inequality of opportunity is accounted for by parental education and occupation. This finding is consistent with Hufe et al. (2023) who identify these components as the strongest drivers of increasing inequality of opportunity in incomes in the US. Second, 29% of the overall increase is accounted for by race. At first glance, this finding appears at odds with the stagnation of racial income gaps since the civil rights era (Bayer and Charles, 2018; Derenoncourt and Mon-

Figure 4. Inequality of Opportunity in the US, 1983-2016
Decomposition by Background Characteristic and Outcome Dimension



(A) Shapley Value Decomposition

(B) Attribute Decomposition

Data: PSID.
Note: This figure shows a decomposition of inequality of opportunity in the US for the *individual sample* over the period 1983-2016. Inequality of opportunity estimates are based on 36 types according to self-reported socio-economic background characteristics. Estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$. The decomposition in Panel (A) is based on the Shapley value procedure proposed in Shorrocks (2013). The decomposition in Panel (B) is based on the attribute decomposition derived in Supplementary Material B.

tialoux, 2021). However, the importance of race increases only after the 2008 financial crisis. Therefore, decreased opportunities to acquire monetary resources are most likely driven by the sustained effect of the financial crisis on the housing wealth of Black Americans (Kuhn et al., 2020; Wolff, 2017). Lastly, the contribution of the region of upbringing remains constant over time.

We also conduct an attribute decomposition, i.e., we decompose the time trend into the contributions of (i) inequality of opportunity in income, (ii) inequality of opportunity in wealth, as well as (iii) the cross-type association in both outcomes. The last dimension is of particular interest as it cannot be analyzed in unidimensional measures of inequality of opportunity. In Supplementary Material B, we show that $I(X)$ can be decomposed as follows:

$$
\begin{aligned}
I(X) = \frac{r_1}{r_1+r_2} & \underbrace{\left(1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1}\right)^{r_1}\right)^{\frac{1}{r_1}}\right)}_{=I_1(\text{Income})} \\
+ \frac{r_2}{r_1+r_2} & \underbrace{\left(1 - \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2}\right)^{r_2}\right)^{\frac{1}{r_2}}\right)}_{=I_2(\text{Wealth})} \\
+ \frac{1}{r_1+r_2} & \underbrace{\left(1 - \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}}\right)}_{=\kappa_I(\text{Association})} \\
+ R, &
\end{aligned}
\tag{4}
$$

where I_q is a unidimensional index of inequality of opportunity in outcome dimension q, κ_I is a measure of cross-type association in outcomes, and R is a residual resulting from linear approximation.[11]

The results of this decomposition are shown in Panel (B) of Figure 4. 43% and 51% of the overall increase in inequality of opportunity can be explained by trends in unidimensional inequality of opportunity in income and wealth, respectively. The cross-type association of income and wealth explains only 6% of the overall increase in unequal opportunities. This finding is somewhat surprising since recent research points to an increased correlation between income and wealth in the US (Berman and Milanovic, 2020; Kuhn and Ríos-Rull, 2016). Our results suggest that these increases at the individual level are mostly driven by increased correlation within family background types while the association of these outcomes across family background types remains rather stable. However, we note that the relative stability of cross-type association κ_I depends on the parameter choices for a_t and r_q. In Supplementary Table S.3, we show the decomposition of time trends under different plausible assumptions for r_q. For example, if we allow for a higher degree of inequality aversion by choosing $r_{Income} = -0.5$ and $r_{Wealth} = -0.5$, κ_I explains up to 40% of the overall increase in unequal opportunities. This finding indicates that family background types in the lower tail of the distribution have become more resource-constrained than the rest of the population in both income and wealth simultaneously.

[11] I_q are unidimensional inequality of opportunity measures based on the Atkinson (1970) index of inequality—see our discussion in section 2.

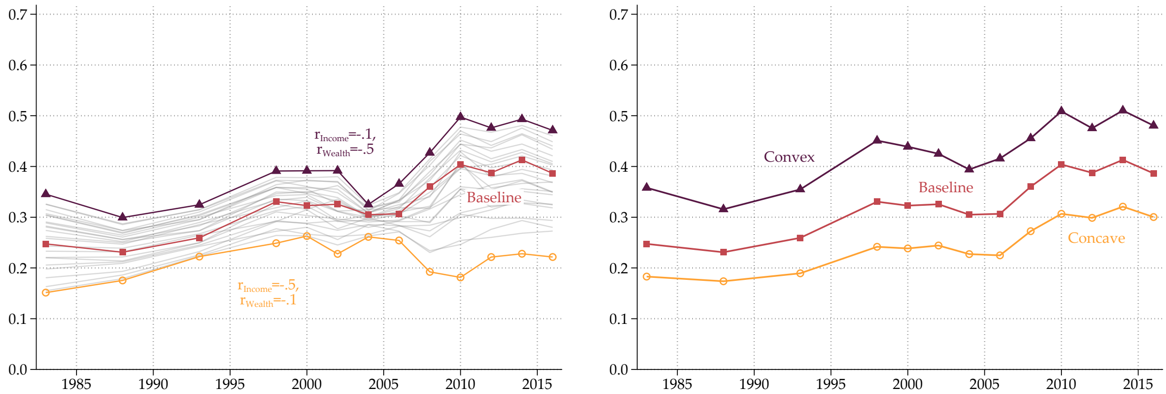
5 SENSITIVITY ANALYSIS

Parameter Choices. We assess the sensitivity of our main conclusions to changes in the measurement parameters, i.e., dimension weights r_q and type weights a_t. Alternative parameter choices correspond to different normative assumptions about inequality aversion. Therefore, they will lead to level shifts in the extent of inequality of opportunity—a property that is well-known in the literature (Atkinson, 1970). However, we are especially concerned with the development of inequality of opportunity over time. In the following discussion, we will therefore abstract from levels and focus on whether changes in unequal opportunities are sensitive to different assumptions about these parameters.

First, dimension weights r_q determine inequality aversion in income and wealth, respectively. In our baseline estimates, we give both dimensions equal weight and choose $r_{Income} = r_{Wealth} = -0.2$. However, there may be good reasons to give different weights to different dimensions of monetary resources. For example, one could argue that wealth should receive a higher weight due to its insurance value. Reversely, one could argue that wealth should receive a lower weight since it is less liquid and might not be available for instantaneous consumption. Panel (A) of Figure 5 shows alternative results for all pairwise combinations over the parameter grid $r_q \in (-0.1, -0.2, -0.3, -0.4, -0.5)$. Lowest estimates of inequality of opportunity are obtained for $r_{Income} = -0.5$ and $r_{Wealth} = -0.1$; that is, in the case where we place little weight on the wealth dimension, and more weight on the income dimension. We note that such income-focused parameterization yields a flat trend after the year 2000. This result is expected and consistent with existing work on intergenerational income mobility (Chetty et al., 2014b; Hartley et al., 2022). However, even small increases in the wealth focus lead to upward corrections in inequality of opportunity estimates and overturn the conclusion of flat time trends after 2000. The highest estimates of inequality of opportunity are obtained for $r_{Income} = -0.1$ and $r_{Wealth} = -0.5$; that is, in the case where we place more weight on the wealth dimension, and little weight on the income dimension.

Second, type weights a_t determine the degree of inequality aversion between types. In our baseline estimates, we choose linear a_t that are inversely related to type ranks in monetary resources. Panel (B) of Figure 5 shows alternative results for convex (a_t^2) and concave type weights ($a_t^{0.5}$). The lowest estimates of inequality of opportunity are obtained for concave type weights, where we place relatively less weight on inequality in the lower tail of the type distribution. Conversely, the highest estimates are obtained for convex type weights, where we place relatively more weight on inequality in the upper tail of the type distribution. Despite changes in levels, our conclusions concerning time trends are insensitive to parameter choices in a_t.

FIGURE 5. Inequality of Opportunity in the US, 1983-2016
Sensitivity to Parameter Choices



(A) Alternations in Dimension Weights (r_q)

(B) Alternations in Type Weights (a_t)

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US for the *individual sample* over the period 1983-2016 under different parameter choices. Panel (A) shows the sensitivity to alternations in r_q. We display are all pairwise combinations of $r_{Income} \in (-0.1, -0.2, -0.3, -0.4, -0.5)$ and $r_{Wealth} \in (-0.1, -0.2, -0.3, -0.4, -0.5)$. The central line replicates our baseline estimates from Figure 3 where we use linear $r_{Income} = r_{Wealth} = -0.2$. Panel (B) shows the sensitivity to alternations in a_t. We construct convex (concave) weights as a_t^2 ($a_t^{0.5}$). The central line replicates our baseline estimates from Figure 3 where we use linear a_t.

Data Choices. In Supplementary Figure S.6, we furthermore document that our main conclusions are robust to different data choices.

First, we recompute inequality of opportunity while smoothing transitory changes in income and wealth, i.e., we replace annual values of income and wealth with 5-year averages. As a consequence, outcome variables provide better proxies for the long-term income and wealth potential of individuals (Solon, 1992). Time trends are very close to our baseline estimates.

Second, we recompute inequality of opportunity for different type partitions. To this end, we code three additional variables and add them to the vector of socioeconomic background characteristics: the number of siblings (11 categories), a dummy for foreign-born parents, and a dummy for single-parent families. In turn, we follow

Brunori et al. (forthcoming) and let a regression tree algorithm decide on the optimal type partition in each year of our analysis. Again, time trends are very similar to our baseline estimates.

Third, we recompute inequality of opportunity for different ways of dealing with non-positive income and wealth. For our baseline, we drop observations with negative income/wealth and set observations with zero income/wealth to 1 USD, respectively. Alternatively, we (i) drop all observations with negative and zero income/wealth, or (ii) retain all observations with negative and zero income/wealth in the sample. Time trends are again very similar, regardless of the chosen specification.

Fourth, we recompute inequality of opportunity using alternative definitions of income and wealth. Our baseline definitions may contain mechanical relationships between income and wealth. Wealth enters household income through capital returns; reversely, savings from household income increase wealth in a given period. Therefore, we divorce both concepts as follows: first, we replace household disposable income with household labor market earnings, i.e., we use an income concept that is not mechanically related to asset returns. Second, we adjust household net worth by deducting active savings in a given year, i.e., we use a wealth concept that is not mechanically related to contemporaneous saving decisions. Our time series are not sensitive to these adjustments, suggesting that mechanical relationships between income and wealth are not the main driver of our results.

We conclude: while the level of inequality of opportunity and the magnitude of its increase varies with different measurement choices, all main conclusions from our baseline estimates remain in place. The only exception arises if we parameterize our index in ways that give little weight to the wealth dimension. In this case, we replicate analyses that focus on the income dimension only and we obtain a flat time trend after the year 2000.

6 CONCLUSION

In this paper, we study inequality of opportunity for the acquisition of monetary resources in the US over the period 1983-2016. In contrast to existing work, we account for the multidimensionality of monetary resources by targeting the joint distribution of income and wealth. Our results show that unidimensional analyses may miss important information when analyzing the playing field in the US: first, we document a more unequal distribution of opportunities when complementing income with the wealth dimension. Second, there are strong and consistent increases in inequality of opportunity over time. This trend is driven by a less opportunity-egalitarian distribution of income until 2000, and a less opportunity-egalitarian distribution of wealth after the financial crisis in 2008.

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Multidimensional Equality of Opportunity in the United States

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Supplementary Material

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A MEASUREMENT—SOME SIMPLE EXAMPLES

Consider a society with two types that are of equal size. Both income and wealth are unequally distributed. Since (i) inequality in both dimensions is exactly the same, and (ii) both dimensions are perfectly correlated across types, inequality of opportunity in monetary resources is exactly the same (0.17) regardless of whether we focus on income (I_{Income}) or wealth (I_{Wealth}) in isolation, or whether we focus on the joint distribution of income and wealth ($I_{Income,Wealth}$).

We now consider three alternative societies in which unidimensional and multidimensional measures of inequality of opportunity diverge. As in the main part of the paper, estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$ and linear a_t that are inversely related to type ranks in monetary resources.

	Income	Wealth
Type 1	50	500
Type 2	100	1000

$$I_{Income} = 0.17$$
$$I_{Wealth} = 0.17$$
$$I_{Income,Wealth} = 0.17$$

(a) (b) (c)

(a)

	Income	Wealth
Type 1	75	500
Type 2	75	1000

$$I_{Income} = 0.00$$
$$I_{Wealth} = 0.17$$
$$I_{Income,Wealth} = 0.09$$

(b)

	Income	Wealth
Type 1	50	1000
Type 2	100	500

$$I_{Income} = 0.17$$
$$I_{Wealth} = 0.17$$
$$I_{Income,Wealth} = 0.06$$

(c)

	Income	Wealth
Type 1	40	1100
Type 2	110	400

$$I_{Income} = 0.27$$
$$I_{Wealth} = 0.27$$
$$I_{Income,Wealth} = 0.12$$

(a) We equalize outcomes across types in the income dimension. Therefore, I_{Income} decreases, and I_{Wealth} stays the same. The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrate the measure's *inequality aversion between types*.

(b) We maintain inequality across types but reverse the cross-type association of income and wealth. Therefore, I_{Income} stays the same, and I_{Wealth} stays the same. The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrate the measure's *sensitivity to correlation-increasing transfers*.

(c) We increase inequality across types in both dimensions and reverse the cross-type association of income and wealth. Therefore, I_{Income} increases, and I_{Wealth} increases. The multidimensional measure $I_{Income,Wealth}$ decreases. This case illustrates the existence of cases where unidimensional and multidimensional measures lead to opposing conclusions. While to former would detect an increase of inequality of opportunity in comparison to the baseline, the latter would detect a decrease in unequal opportunities.

B ATTRIBUTE DECOMPOSITION

In this appendix, we derive and prove the attribute decomposability of $I(X)$ as defined in Equation (3). Our derivation is based on results presented in Abul Naga and Geoffard (2006). For the exposition, we focus on the case of two outcome dimensions with $K = 2$.[1] In this case, X consists of two submatrices X_1 and X_2 that denote outcome matrices for dimensions 1 and 2, respectively. Recall that μ_q^t denotes a type mean in outcome dimension q. Given the notation with two submatrices, an element x_{i1} (x_{i2}) of matrix X_1 (X_2) equals μ_1^t (μ_2^t), i.e, the mean value of dimension 1 (dimension 2) in type t to which individual i belongs. Finally, recall that μ_q denotes the population mean of dimension q.

Attribute Decomposability. In general, $I(X) = 1 - \delta(X)$, where $\delta(X) \in [0,1)$. $I(X)$ is attribute decomposable if and only if

$$\delta(X) = f_1(\gamma_1(X_1)) + f_2(\gamma_2(X_2)) + f_3(\kappa(X)), \tag{5}$$

where f_1, f_2, f_3 are increasing functions ($\mathbb{R}_+ \mapsto \mathbb{R}_+$), γ_1 and γ_2 are unidimensional equality indices, and κ is a measure of association between X_1 and X_2.

Proposition 1. *$\delta(X)$ is attribute decomposable as follows:*

$$\ln \delta(X) = \frac{r_1}{r_1 + r_2} \ln \gamma_1(X_1) + \frac{r_2}{r_1 + r_2} \ln \gamma_2(X_2) + \frac{1}{r_1 + r_2} \ln \kappa(X), \tag{6}$$

[1] We note this restriction can be easily relaxed.

where

$$\gamma_1(X_1) = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1} \right)^{r_1} \right)^{\frac{1}{r_1}},$$

$$\gamma_2(X_2) = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2} \right)^{r_2} \right)^{\frac{1}{r_2}},$$

$$\kappa(X) = \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}}.$$

Proof. First, $\delta(X)$ is the proportion of μ_q that is necessary to achieve the same level of welfare if all attributes were distributed equally across types, see Kobus et al. (2020). Formally, let $w_0 = \sum_{t=1}^{M} N_t U^t(\delta\mu_1, \delta\mu_2)$ denote the welfare level associated with X. Second, let ρ_1 be the proportion of μ_1 that is necessary to attain w_0, if (i) the first attribute was equally distributed across types, and (ii) the distribution of the second attribute across types remained as is. Formally, $w_0 = \sum_{t=1}^{M} N_t U^t(\rho_1\mu_1, \mu_2^t)$. Third, let γ_1 be the proportion of μ_1 that is necessary to attain w_0, if (i) the first attribute was equally distributed across types, and (ii) the second attribute was equally distributed across types. Formally, $w_0 = \sum_{t=1}^{M} N_t U^t(\gamma_1\mu_1, \rho_2\mu_2)$.

It follows that

$$w_0 = \sum_{t=1}^{M} N_t a_t (\delta\mu_1)^{r_1} (\delta\mu_2)^{r_2} = \sum_{t=1}^{M} N_t a_t (\gamma_1\mu_1)^{r_1} (\rho_2\mu_2)^{r_2}.$$

After modification, we get $\delta^{r_1+r_2} = (\gamma_1)^{r_1}(\rho_2)^{r_2}$, and we obtain

$$\ln(\delta) = \frac{r_1}{r_1+r_2}\ln(\gamma_1) + \frac{r_2}{r_1+r_2}\ln(\gamma_2) + \frac{1}{r_1+r_2}\ln\left(\rho_2/\gamma_2\right)^{r_2}, \tag{7}$$

which is the desired decomposition with $\kappa := (\rho_2/\gamma_2)^{r_2}$.

We now need to derive functional forms of γ_1, γ_2 and κ.

Note that $w_0 = \sum_{t=1}^{M} N_t a_t (\gamma_1\mu_1)^{r_1} (\rho_2\mu_2)^{r_2} = \sum_{h=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\rho_2\mu_2)^{r_2}$. Solving for γ_1 yields:

$$\gamma_1 = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_1^t}{\mu_1} \right)^{r_1} \right)^{\frac{1}{r_1}}.$$

Proceeding in analogy, for γ_2 we get:

$$\gamma_2 = \left(\sum_{t=1}^{M} \frac{N_t a_t}{\sum_{t=1}^{M} N_t a_t} \left(\frac{\mu_2^t}{\mu_2} \right)^{r_2} \right)^{\frac{1}{r_2}}.$$

Furthermore, we use $w_0 = \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\rho_2 \mu_2)^{r_2} = \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}$ to obtain

$$\rho_2 = \left(\frac{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2)^{r_2}} \right)^{\frac{1}{r_2}}.$$

Finally, substituting the expressions for γ_2 and ρ_2 into $\kappa := (\rho_2/\gamma_2)^{r_2}$ we get:

$$\kappa = \frac{\sum_{t=1}^{M} N_t a_t \sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} (\mu_2^t)^{r_2}}{\sum_{t=1}^{M} N_t a_t (\mu_1^t)^{r_1} \sum_{t=1}^{M} N_t a_t (\mu_2^t)^{r_2}}.$$

□

Linear Approximation. Collecting terms and reversing the log-linearization of $\delta(X)$, we obtain the attribute decomposition of $I(X)$ displayed in Equation (5):

$$I(X) = 1 - (\gamma_1)^{\frac{r_1}{r_1+r_2}} (\gamma_2)^{\frac{r_2}{r_1+r_2}} (\kappa)^{\frac{1}{r_1+r_2}}. \tag{8}$$

Applying a linear approximation around the point of perfect equality (i.e., $\gamma_1 = \gamma_2 = \kappa = 1$), we get the linear decomposition displayed in Equation (4):

$$\begin{aligned} I(X) &= \tfrac{r_1}{r_1+r_2}(1-\gamma_1) + \tfrac{r_2}{r_1+r_2}(1-\gamma_2) + \tfrac{1}{r_1+r_2}(1-\kappa) + R, \\ &= \tfrac{r_1}{r_1+r_2} I_1 + \tfrac{r_2}{r_1+r_2} I_2 + \tfrac{1}{r_1+r_2}\kappa_I + R. \end{aligned} \tag{9}$$

C ADDITIONAL FIGURES AND TABLES

TABLE S.1. Descriptive Statistics

	Income	Wealth	Family Background				Age	N
			Educ.	Occ.	Race	Region		
Panel (A): Intergenerational Sample								
	55,745	279,508	2.26	2.32	0.87	0.26	50	1,366
Panel (B): Re-weighted Intergenerational Sample								
	49,150	205,851	2.20	2.28	0.75	0.34	46	1,366
Panel (C): Individual Sample								
1983	34,763	160,272	1.75	1.87	0.84	0.32	41	5,368
1988	42,258	171,767	1.87	1.94	0.82	0.31	40	5,357
1993	40,562	166,160	1.95	2.00	0.81	0.31	41	5,070
1998	44,563	183,894	2.04	2.09	0.79	0.37	42	4,213
2000	46,136	200,777	2.06	2.12	0.78	0.37	43	4,106
2002	46,414	199,987	2.05	2.13	0.77	0.38	43	4,238
2004	48,897	228,384	2.05	2.14	0.77	0.36	43	5,197
2006	49,406	254,948	2.07	2.15	0.76	0.36	43	5,250
2008	48,349	214,078	2.08	2.16	0.76	0.36	44	5,079
2010	45,490	189,976	2.09	2.18	0.73	0.35	44	5,039
2012	46,377	167,962	2.10	2.19	0.72	0.36	44	5,047
2014	46,373	178,185	2.12	2.20	0.71	0.35	44	5,013
2016	46,837	188,240	2.12	2.21	0.70	0.35	44	4,957

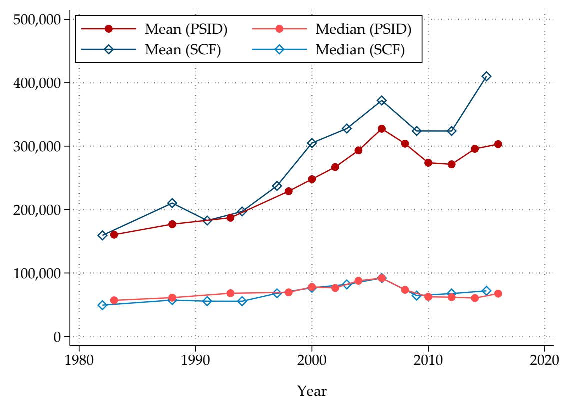
Data: PSID.
Note: This table displays summary statistics for the *intergenerational sample* (Panel [A]), the re-weighted *intergenerational sample* (Panel [B]) and the *individual sample* (Panel [C]). Income is defined as annual disposable household income, wealth as household net worth. Both income and wealth are scaled by the modified OECD equivalence scale and expressed in constant 2015 USD. We furthermore drop observations with negative income/wealth and set zero amounts to 1 USD. The family background variables Educ. (Occ.) show the average education (occupation) level of the parent with the highest education (occupation) status measured on a 3-point scale. Race displays the share of whites; region the share of respondents who grew up in the US Census region South. Age refers to the average age in the sample. The last column shows the number of observations.

TABLE S.2. Observation Loss due to Sample Restrictions

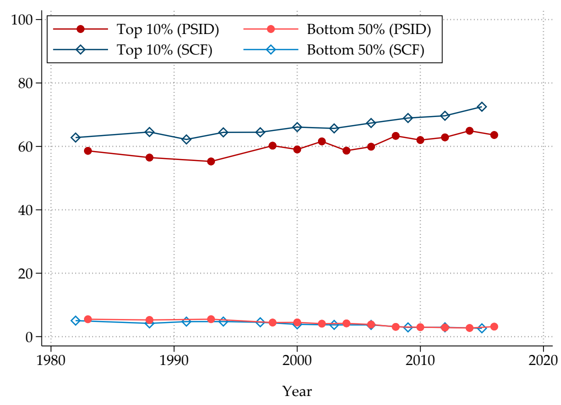
Time Period	Sample Restriction	Observations	Share of Original Sample
Panel (A): Intergenerational Sample			
2010-2016		8,824	100%
2010-2016	(i)	8,061	91.4%
2010-2016	(ii)	7,187	89.3%
2010-2016	(iii)	1,366	15.5%
Panel (B): Individual Sample			
1983-2016		80,918	100%
1983-2016	(i)	69,267	81.4%
1983-2016	(ii)	63,934	79.0%
1983		6,257	100%
1983	(i)	5,732	91.6%
1983	(ii)	5,368	85.8%
1993		6,012	100%
1993	(i)	5,384	89.6%
1993	(ii)	5,070	84.3%
2004		6,392	100%
2004	(i)	5,610	87.8%
2004	(ii)	5,197	81.3%
2014		6,896	100%
2014	(i)	5,569	80.8%
2014	(ii)	5,013	72.7%

Data: PSID.
Note: This table shows the loss in observations when defining our analysis samples. For the *intergenerational sample*, we require (i) non-missing and non-negative information on income and wealth, and (ii) non-missing information on parental education, parental occupation, race, and region of upbringing. We further require that (iii) information on the income of both parents is non-missing. For the *individual sample*, we require (i) and (ii) only.

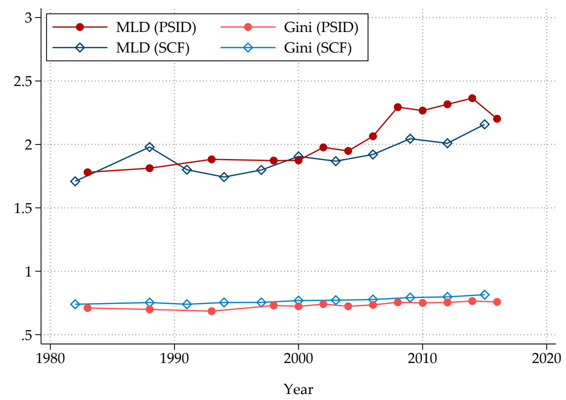
FIGURE S.1. Wealth in PSID and SCF, 1983-2016



(A) Mean and Median (in USD)



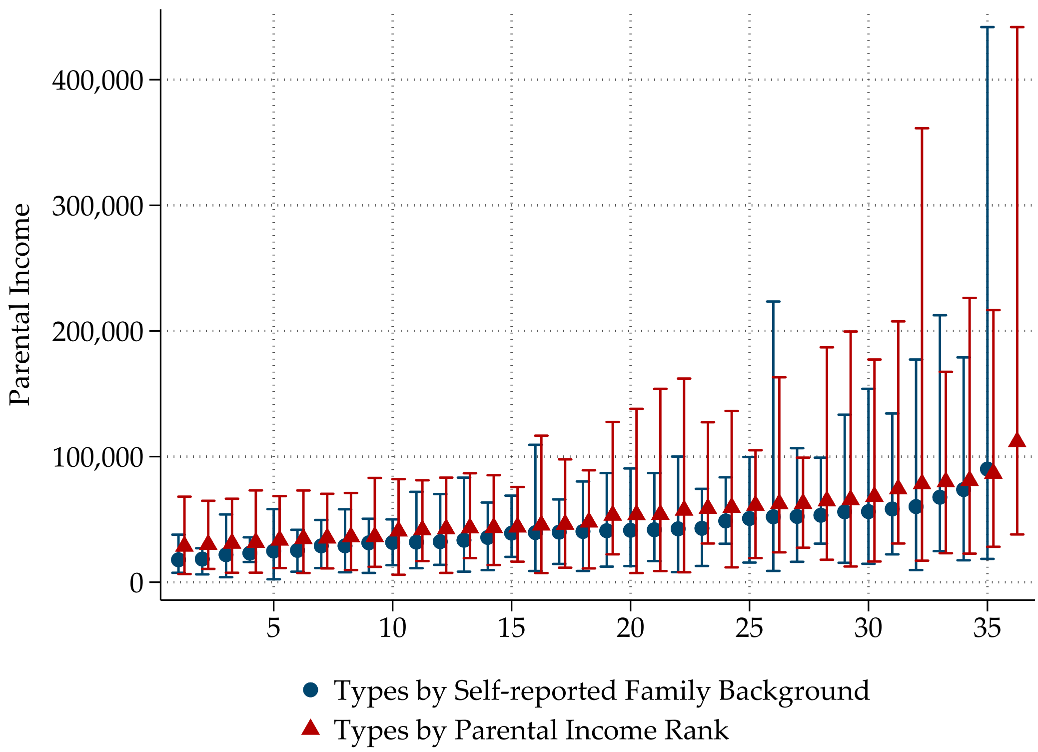
(B) Income Shares at Bottom and Top (in %)



(C) Inequality

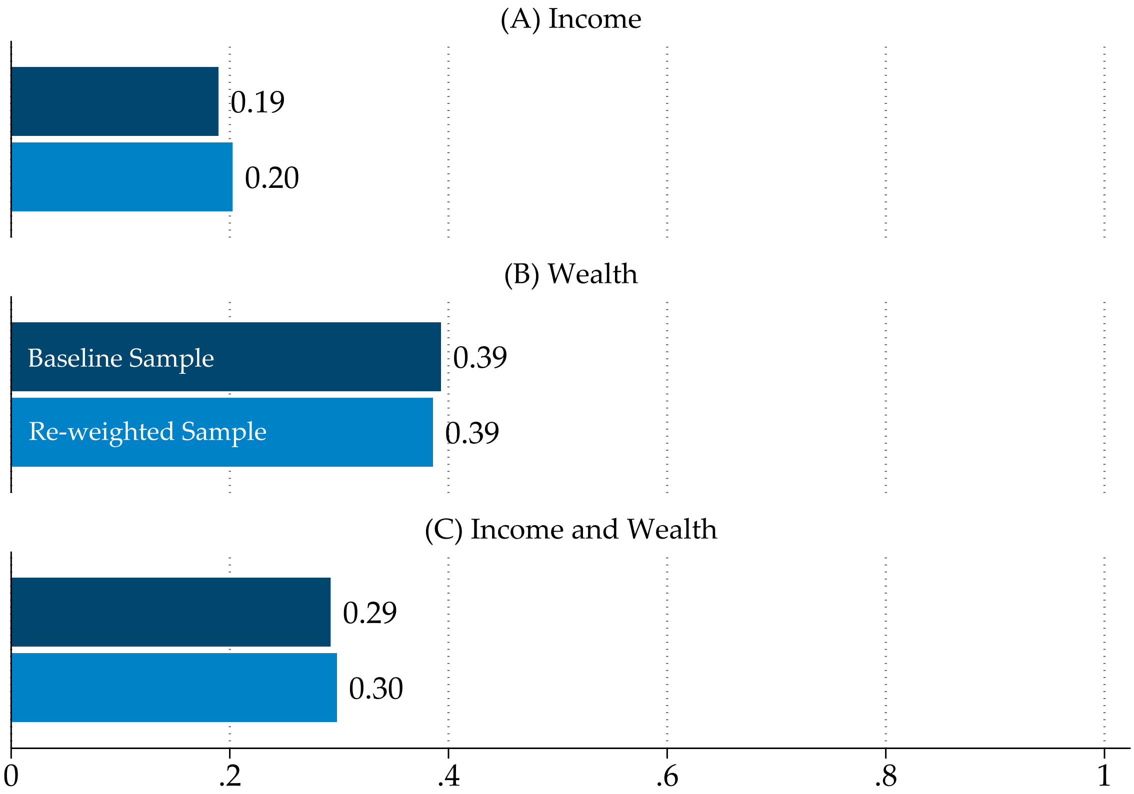
Data: PSID, SCF+ (Kuhn et al., 2020).
Note: This figure compares wealth distributions between the PSID and the Survey of Consumer Finances (SCF). In both data sources, wealth is defined as equivalized household net worth (see Section 3); we drop negative values, replace zero values with 1 USD, and winsorize from above at the 99.9 percentile. Samples are restricted to household heads. All figures are expressed in constant 2015 USD.

FIGURE S.2. Parental Income by Family Background Type



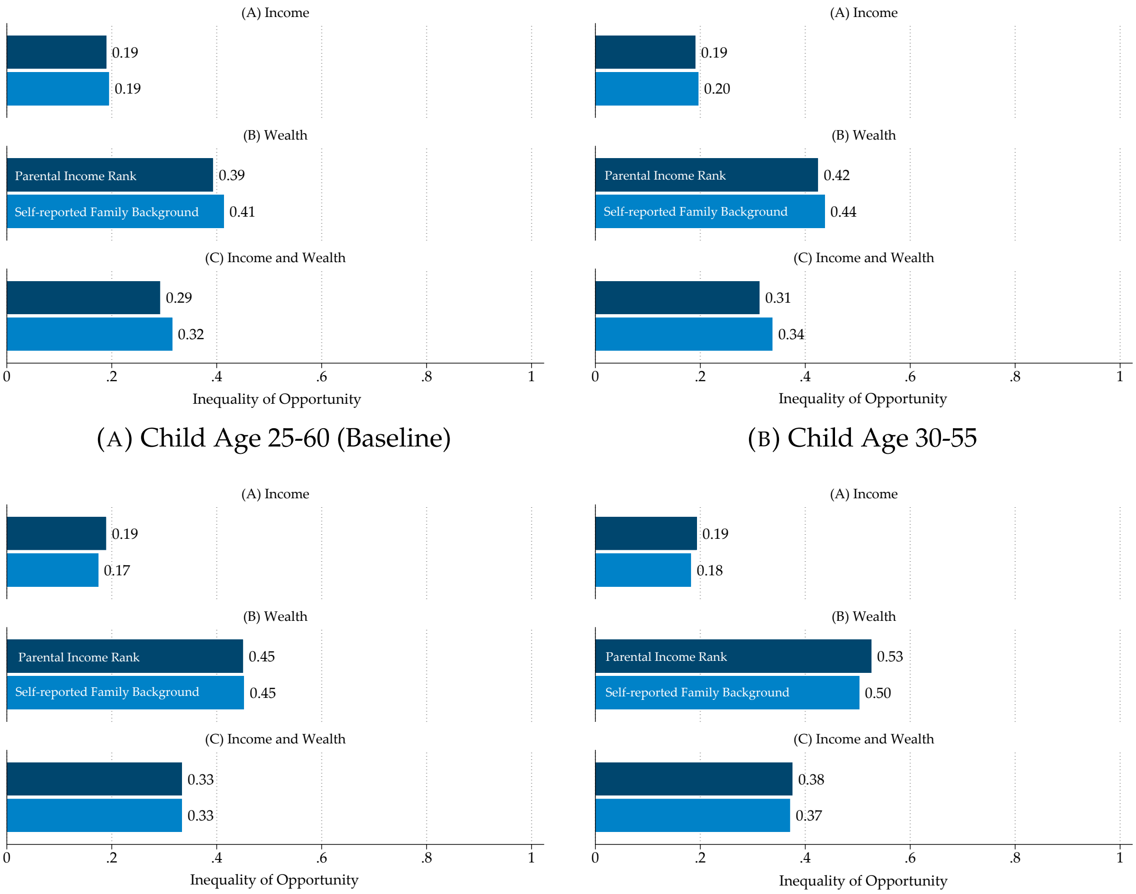
Data: PSID.
Note: This figure shows the mean, 5th-percentile, and 95th-percentile of income distributions within different types of family background characteristics in the *intergenerational sample*. Blue circles and whiskers refer to self-reported family background characteristics. Red triangles and whiskers refer to parental income ranks. Note that bins of family background characteristics tend to be of unequal size, whereas bins of parental income ranks tend to be of equal size (in absence of ties). This feature and resulting differences in the weighting of types explain that averages based on parental income ranks tend to be slightly higher than corresponding averages based on self-reported family background characteristics although the underlying income distributions are the same.

FIGURE S.3. Inequality of Opportunity in the US
Re-weighted Sample



Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US when accounting for selective sample attrition in the *intergenerational sample*. In particular, we re-weight the *intergenerational* sample to match the *individual sample* in the observation period 2010-2016 concerning age, parental education, parental occupation, race, and region of upbringing. All estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$ and use parental income rank as a proxy for family background.

FIGURE S.4. Equality of Opportunity in the US
Varying Age Restrictions



(A) Child Age 25-60 (Baseline)

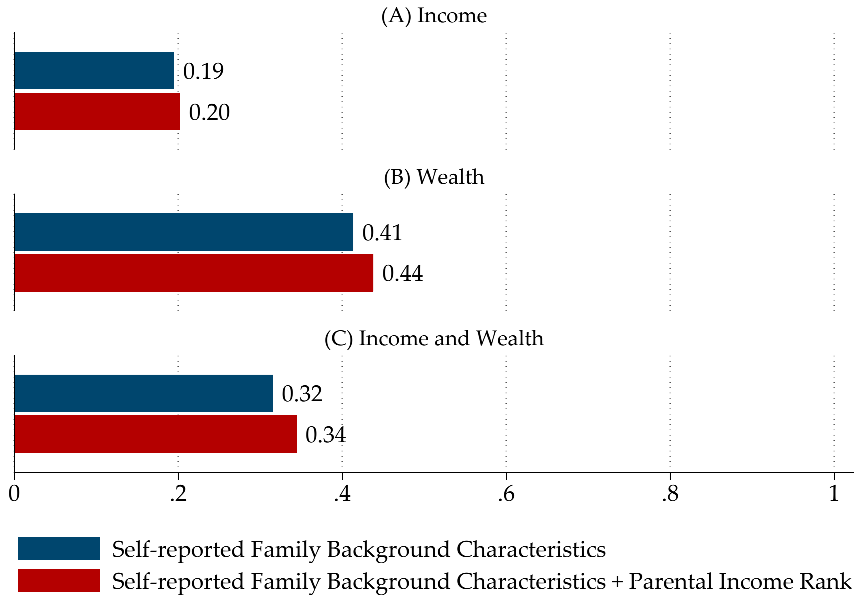
(B) Child Age 30-55

(C) Child Age 35-50

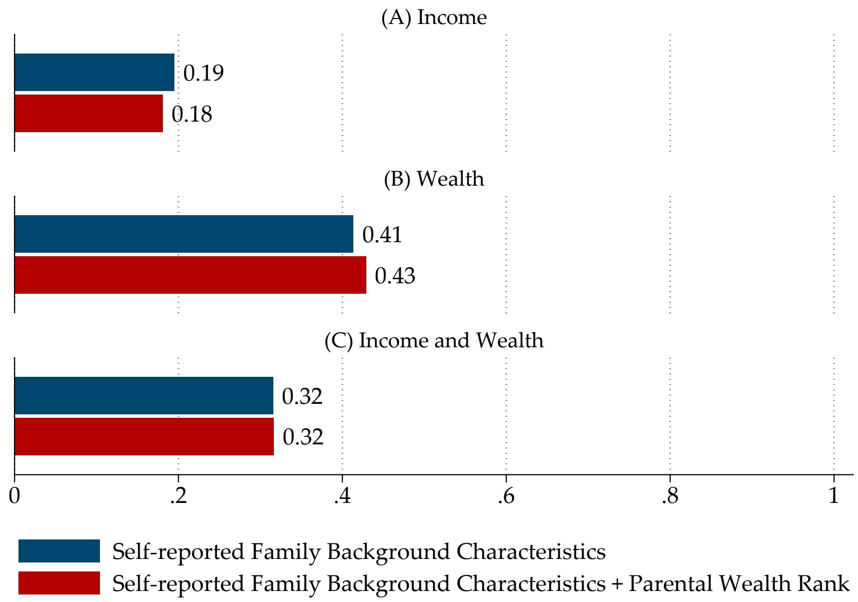
(D) Child Age 40-45

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US to different sample restrictions regarding the age of children. Panel (A) replicates our baseline estimates from Figure 2. In Panels (B)-(D) we sequentially narrow the age restriction to 30-55, 35-50, and 40-45. All estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

Figure S.5. Equality of Opportunity in the US
Extended Family Background Characteristics



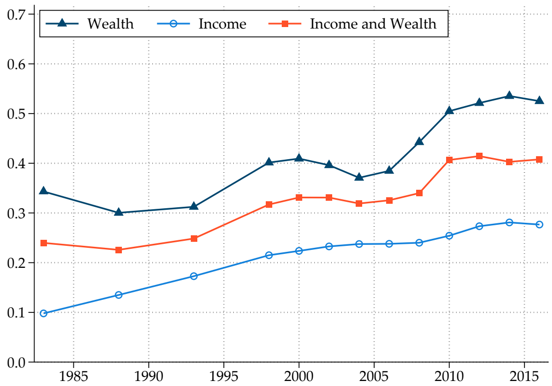
(A) Adding Parental Income Ranks



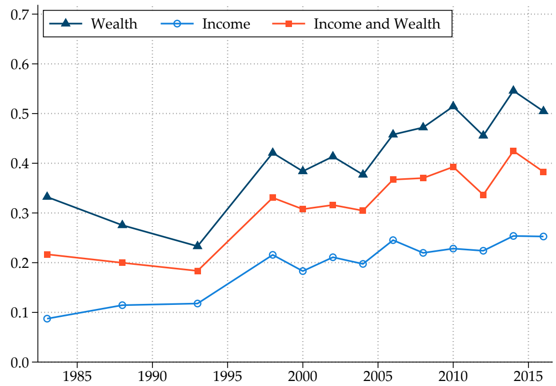
(B) Adding Parental Wealth Ranks

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US to extended sets of eligible family background characteristics. The blue bars replicate our baseline estimates from Figure 2 based on self-reported family background characteristics (parental education, parental occupation, race, region of upbringing). The red bars show estimates when adding 100 parental income ranks (Panel A) or 100 parental wealth ranks (Panel B) to self-reported family background characteristics and selecting types via a regression tree algorithm. All estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

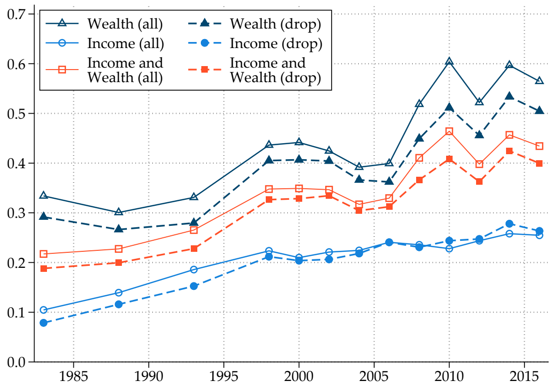
FIGURE S.6. Inequality of Opportunity in the US, 1983-2016
Sensitivity to Data Choices



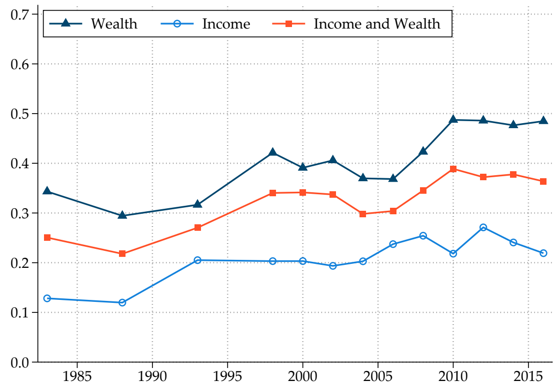
(A) 5-Year Averages



(B) Alternative Type Partition



(C) Accounting for Negative Values



(D) Labor Income and Wealth Net of Active Savings

Data: PSID.
Note: This figure shows the sensitivity of inequality of opportunity in the US for the *individual sample* over the period 1983-2016. In Panel (A), we take a 5-year moving average of income and wealth. In Panel (B), we let a regression tree determine the underlying type partition. In Panel (C), we keep zero income and wealth without adjustment (solid line) or drop individuals with zero income or wealth (dashed line). Panel (D) displays our estimates for the sub-components of labor income and wealth net of active savings in the period of interest. Estimates are computed based on Equation (3) with dimension weights $r_{Income} = r_{Wealth} = -0.2$.

TABLE S.3. Attribute Decomposition
Alternative Parameterization

r_{Income}	r_{Wealth}	Contribution of		
		Income	Wealth	Association
-0.1	-0.1	42%	54%	2%
-0.2	-0.2	43%	51%	6%
-0.3	-0.3	43%	48%	12%
-0.4	-0.4	44%	46%	17%
-0.5	-0.5	46%	34%	40%

Data: PSID.
Note: This table displays the relative contribution of I_{Income}, I_{Wealth}, and κ_I to the increase in multidimensional inequality of opportunity over the time period 1983-2016. The decomposition is based on the attribute decomposition derived in Supplementary Material B.

References

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